Problem 12-7

A particle moves along a straight line such that its position is defined by $s = (t^2 - 6t + 5)$ m. Determine the average velocity, the average speed, and the acceleration of the particle when $t = 6$ s.

Solution

The average velocity when $t = 6$ s is

$$v_{\text{avg}} = \frac{\Delta s}{\Delta t} = \frac{s(6) - s(0)}{(6) - (0)} = \frac{[(6)^2 - 6(6) + 5] - [0^2 - 6(0) + 5]}{6} = 0 \ \frac{\text{m}}{\text{s}}.$$

Differentiate the given position function to determine the velocity.

$$\begin{aligned} v &= \frac{ds}{dt} \\ &= \frac{d}{dt}(t^2 - 6t + 5) \\ &= 2t - 6 \end{aligned}$$

Find the total distance that the particle travels from $t = 0$ s to $t = 6$ s.

$$\begin{aligned} s_T = \int_0^6 |v(t)|\, dt &= \int_0^6 |2t - 6|\, dt \\ &= \int_0^3 (6 - 2t)\, dt + \int_3^6 (2t - 6)\, dt \\ &= (6t - t^2)\Big|_0^3 + (t^2 - 6t)\Big|_3^6 \\ &= (9) + (9) \\ &= 18 \text{ m} \end{aligned}$$

The average speed is then

$$\frac{s_T}{\Delta t} = \frac{18 \text{ m}}{(6 - 0) \text{ s}} = 3 \ \frac{\text{m}}{\text{s}}.$$

Differentiate the velocity function to determine the acceleration.

$$\begin{aligned} a &= \frac{dv}{dt} \\ &= \frac{d}{dt}(2t - 6) \\ &= 2 \end{aligned}$$

Therefore, the acceleration of the particle at $t = 6$ s is

$$a(6) = 2 \ \frac{\text{m}}{\text{s}^2}.$$